NASA TECHNICAL TRANSLATION

NASA TT F-363

MOTION OF A PENDULUM WITH STRONG DAMPING ON A VIBRATING BASE

by D. M. Klimov

Izvestiya Akademii Nauk SSSR
Mekhanika i Mashinostroyeniye, No. 4, 1964

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MAY 1965

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Translation of "O dvizhenii mayatnika s sil'nym dempfirovaniyem na vibriruyushchem osnovanii."
Izvestiya Akademii Nauk SSSR. Mekhanika i Mashinostroyeniye.
No. 4, 1964, pp. 142-143.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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Springfield, Virginia 22151 – Price $1.00

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(Moscow)

ABSTRACT

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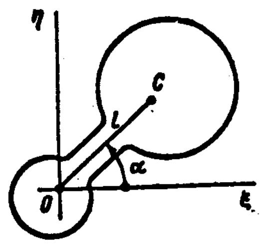
The motion of a pendulum with its axis oriented vertically is investigated. It is shown that vibration of the base gives rise to a slow unidirectional rotation on the part of the pendulum. The mean angular velocity of this rotation is calculated.

Author

To the base on which the pendulum is mounted we affix a translationally moving coordinate system $\xi\eta\zeta$. We align the ζ-axis along the vertically oriented axis of rotation of the pendulum (this axis is perpendicular to the plane of the paper in the figure). The angle through which the pendulum rotates is denoted by l, the distance from the center of gravity C to the rotational axis by α. The equation of motion of the pendulum has the form

$$A\frac{d^2\alpha}{dt^2}+h\frac{d\alpha}{dt}=mlW_\xi\sin\alpha-mlW_\eta\cos\alpha \qquad (1)$$



Here, A is the moment of inertia of the pendulum relative to the rotational axis, h is the damping factor, presumed large, m is the mass of the pendulum, W_ξ and W_η are the projections of the base acceleration.

Let the acceleration of the base due to vibration vary according to the law

$$W_\xi=a_1\omega^2\sin\omega t,\qquad W_\eta=a_2\omega^2\sin(\omega t+\theta) \qquad (2)$$

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where θ is some constant phase shift. We introduce the following notation:

$$a_1\omega^2/g = n_1, \quad a_2\omega^2/g = n_2, \quad mgln_1 = p_1, \quad mgln_2 = p_2 \tag{3}$$

Here g is the free-fall acceleration, n_1 and n_2 are the vibration overloads. Taking (2) and (3) into account, equation (1) assumes the form

$$A\frac{d^2\alpha}{dt^2} + h\frac{d\alpha}{dt} = p_1 \sin\alpha \sin\omega t - p_2 \cos\alpha \sin(\omega t + \theta) \tag{4}$$

In the latter equation, we perform the substitution $\alpha = \alpha_0 + x$ (where α_0 is the value of the angle α at the initial instant) and consider x to be small, i.e., we let $\cos x \approx 1$, $\sin x \approx x$.

We have

$$\begin{aligned} A\frac{d^2x}{dt^2} + h\frac{dx}{dt} = p_1(\sin\alpha_0 + x\cos\alpha_0)\sin\omega t - \\ - p_2(\cos\alpha_0 - x\sin\alpha_0)(\sin\omega t\cos\theta + \cos\omega t\sin\theta) \end{aligned} \tag{5}$$

In equation (5), we make the following change of variables:

$$\begin{aligned} x &= m_1\sin\omega t + m_2\cos\omega t + x_1 \\ m_1 &= -\frac{hp_2\cos\alpha_0\sin\theta + A\omega(p_1\sin\alpha_0 - p_2\cos\alpha_0\cos\theta)}{\omega(A^2\omega^2 + h^2)} \\ m_2 &= \frac{A\omega p_2\cos\alpha_0\sin\theta - h(p_1\sin\alpha_0 - p_2\cos\alpha_0 - \cos\theta)}{\omega(A^2\omega^2 + h^2)} \end{aligned} \tag{6}$$

We obtain

$$A\frac{d^2x_1}{dt^2} + h\frac{dx_1}{dt} = f(x_1, \omega t) \tag{7}$$

Here

$$\begin{aligned} f(x_1, \omega t) = [(p_1\cos\alpha_0 + p_2\sin\alpha_0\cos\theta)\sin\omega t + \\ + p_2\sin\alpha_0\sin\theta\cos\omega t](m_1\sin\omega t + m_2\cos\omega t + x_1) \end{aligned} \tag{8}$$

We seek the solution to equation (7) in the form

$$x_1 = C_1 + C_2e^{-kt} \quad (k = h/A), \qquad dx_1/dt = -kC_2e^{-kt} \tag{9}$$

The unknown variables C_1 and C_2 satisfy the relation

$$dC_1/dt + e^{-kt}dC_2/dt = 0 \tag{10}$$

From equation (7), bearing in mind (9) and (10), we find

$$he^{-kt}\, dC_2/dt = -f(x_1, \omega t), \qquad h\, dC_1/dt = f(x_1, \omega t) \tag{11}$$

Let us examine the case in which the base motion is such that the vibration frequency is large and the parameters p_1 and p_2 (see eq. (3)) are small. /143

We will confine our analysis to motion of the pendulum during a time interval in which x_1 varies only negligibly. It is apparent from equation (8) that in this case, $f(x_1, \omega t)$ is small.

Equations (11), taking into account the assumptions made above, shows that the functions C_1 and C_2e^{-kt} vary slowly with time.

Consequently, the function $f(C_1 + C_2e^{-kt}, \omega t)$ can be replaced in equations (11) by its mean value (ref. 1), averaging over ωt and, in so doing, treating C_1 and C_2e^{-kt} as constants. The mean value f_0 of the function $f(C_1 + C_2e^{-kt}, \omega t)$ is determined by straightforward calculations. We have

$$\begin{aligned} 2f_0 = \langle 2\,[(p_1 \cos \alpha_0 + p_2 \sin \alpha_0 \cos \theta) \sin \omega t + p_2 \sin \alpha_0 \sin \theta \cos \omega t]\,(m_1 \sin \omega t + \\ + m_2 \cos \omega t + C_1 + C_2 e^{-kt})\rangle = m_1 (p_1 \cos \alpha_0 + p_2 \sin \alpha_0 \cos \theta) + m_2 p_2 \sin \alpha_0 \sin \theta \end{aligned} \tag{12}$$

Here and below, the angular brackets $\langle \ldots \rangle$ denote the time average of the bracketed quantity.

Replacing the function $f(C_1 + C_2e^{-kt}, \omega t)$ in the first equation of the system (11) by its mean value f_0 and integrating, we obtain

$$C_2 = -\frac{f_0}{kh} e^{kt} + C_2(0)$$

We now substitute the variable C_2 into equation (9). We have

$$\frac{dx_1}{dt} = \frac{f_0}{h} - k\, C_2(0)\, e^{-kt}$$

Hence

$$\left\langle \frac{d\alpha}{dt} \right\rangle = \left\langle \frac{dx_1}{dt} \right\rangle = \frac{f_0}{h};$$

Finally, bearing in mind equations (6) and (12), we obtain the equation for the mean angular velocity of the pendulum in the form

$$\left\langle \frac{da}{dt} \right\rangle = \frac{A\,(p_2^2 - p_1^2)\,\sin 2a_0}{4h\,(A^2\omega^2 + h^2)} + \frac{Ap_1p_2 \cos 2a_0 \cos\theta}{2h\,(A^2\omega^2 + h^2)} - \frac{p_1p_2 \sin\theta}{2\omega\,(A^2\omega^2 + h^2)}$$

It is apparent from the latter equation that vibration of the base can elicit rotation of the pendulum in one direction. Specifically, if the moment of inertia of the pendulum is sufficiently small, such rotation proceeds at a constant angular velocity:

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{p_1p_2 \sin\theta}{2\omega h^2}$$

Received 18 March 1964

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